

**DETERMINATION** > FINANCIAL ARRANGEMENTS > GENERAL

# Present Value Based Yield to Maturity Method

Issued: 24 October 1990

G11A

This Determination may be cited as "Determination G11A: Present Value Based Yield to Maturity Method".

# 1 Explanation (which does not form part of the determination)

1. This determination rescinds and replaces Determination G11: Present Value Based Yield to Maturity Method made by the Commissioner on 21 November 1988. This determination differs from Determination G11 by the addition of an example of its application to perpetuities in which all payments are the same after some period.
2. This Determination states how the yield to maturity method shall be applied to a financial arrangement to calculate income derived or expenditure incurred for purposes of Section 64C of the Income Tax Act 1976.
3. This Determination is an alternative to Determination G3: Yield to Maturity Method and will give very similar answers when used with Method A of Determination G10B: Present Value Calculation Methods.
4. The Determination applies to any financial arrangement where all the amounts and dates are known (or in the case of perpetuities the amounts of all payments due after a certain date, and the intervals at which they are payable, are able to be determined) not later than the first balance date of the issuer or holder after issue or acquisition, as the case may be, and determined in a single currency.
5. The approach adopted is to define a constant annual interest rate representing the yield to maturity of all the cash flows in the financial arrangement. Income derived and expenditure incurred is assumed to be compounded on the date of each payment. The calculations are simplified by using regular periods such as half years, months or weeks, where most of all of the cash flows occur at such intervals. However, where a period between payments is longer than one year, income derived and expenditure incurred must be compounded at yearly intervals.
6. In general, there is no explicit formula for a yield to maturity in terms of the cash flows. For purposes of this Determination, the yield to maturity is defined as the interest rate at which the Present Value of all amounts payable after the date of issue or acquisition is equal to the amount payable on that date.
7. The formulae and method for calculating the Present Value will depend on the nature of the financial arrangement and a number of alternatives will be published in Determination G10B: Present Value Calculation Methods for this purpose.
8. Persons may use either Determination G3: Yield to Maturity Method or this Determination for the purpose of applying the yield to maturity method to a financial arrangement. Once the person has elected to use Determination G3: Yield to Maturity

Method or this Determination for a particular financial arrangement, the method set out in that Determination shall be used by the person over the life of that financial arrangement, unless the prior consent of the Commissioner is obtained to change methods.

9. At each balance date after the date of issue or acquisition, the Present Value of the arrangement is recalculated using the same formula and method as was used originally to calculate yield to maturity (see paragraph (6) above).
10. It will be noted that in the case of perpetuities there may not ever be a base price adjustment (see Example D).
11. The amount of income derived or expenditure incurred by the holder or issuer in respect of the Income Year ending on that balance date will be—
  - (a) The Present Value at the balance date,
  - less (b) Any amounts payable by the holder or receivable by the issuer (as the case may be) during the Income Year,
  - plus (c) Any amounts payable by the issuer or receivable by the holder (as the case may be) during the Income Year,
  - less (d) The Present Value as at the preceding balance date (or date of issue or acquisition if later).

In most normal commercial financial arrangements, the yield to maturity will be unique, positive and less than 100 percent per annum; arrangements that do not meet these criteria are excluded and may be submitted for individual consideration by the Commissioner.

## 2 Reference

1. This Determination is made pursuant to Section 64E(1)(a) and section 64E(6) of the Income Tax Act 1976.
2. Determination G11: Present Value Based Yield to Maturity Method is hereby rescinded with effect from the day on which this Determination G11A is signed.

## 3 Scope of Determination

This Determination applies to the class of financial arrangements that has the following attributes—

- (a) All amounts payable or receivable under the financial arrangement, and the dates on which those amounts are payable or receivable, are known (or, in the case of perpetuities, able to be determined) not later than the first balance date of the issuer or holder following the date of issue or acquisition, as the case may be; and
- (b) All amounts are determined in a single currency,

but does not apply to any financial arrangement in respect of which the Annual Yield To Maturity Rate as defined in clause 5 of this Determination would be—

- (a) Not unique; or
- (b) Less than zero; or
- (c) Greater than 100 percent.

## 4 Principle

The yield to maturity method apportions the total income or expenditure under a financial arrangement between income years so that the principal outstanding and accrued interest calculated as at any balance date represents the then Present Value of all amounts payable after that balance date, calculated in accordance with the formulae and method used to calculate the yield to maturity rate as at the date of issue or acquisition of the financial arrangement.

## 5 Interpretation

1. In this Determination, unless the context otherwise requires—

“Annual Yield To Maturity Rate”, in relation to a person and a financial arrangement, means the constant annual rate of interest at which the Present Value of all amounts payable to and by the person after the date of acquisition or issue of the financial arrangement, calculated as at that date, is equal to the net amount payable to or by the person on that date;

“Income Year” has the same meaning as in Sections 64B to 64M of the Act;

“Present Value”, in relation to a person and a financial arrangement and a day, means the discounted value on that day of all amounts payable to or by the person after that day, calculated in accordance with a formula and method specified in Determination G10B: Present Value Calculation Methods, the amounts being discounted at a constant annual rate of interest; and for this purpose the Present Value of a financial arrangement shall be calculated using the Annual Yield To

Maturity Rate in accordance with the formula and method that were used in the calculation of the Annual Yield To Maturity Rate of that financial arrangement.

2. For convenience, words and phrases defined in this Determination are indicated by initial capital letters, but the absence of a capital letter shall not alone imply that the word or phrase is used with a meaning different from that given by its definition.
3. In this Determination, unless the context otherwise requires, expressions used that are not defined in clause 5(1) have the same meanings as in Sections 2 and 64B of the Income Tax Act 1976.
4. Any reference in this Determination to another Determination made by the Commissioner shall be construed as referring to any fresh Determination made by the Commissioner to vary, rescind, restrict or extend that Determination.

## 6 Method

1. This Determination may, at the election of the taxpayer, be used instead of Determination G3: Yield to Maturity Method. Once a taxpayer has made the election to use this Determination to apply the yield to maturity method to a financial arrangement the taxpayer shall then apply this Determination in respect of the financial arrangement until it matures or is remitted, sold or otherwise transferred by the taxpayer unless the prior consent of the Commissioner is obtained to change methods. Any such consent may be given conditionally.
2. The amount of income derived or expenditure incurred by the holder or issuer in respect of any Income Year (other than an Income Year to which Section 64F of the Income Tax Act 1976 applies) shall be—
  - (a) The Present Value of the financial arrangement as at the balance date on which the Income Year ends,
  - less (b) Any amounts payable by the holder or receivable by the issuer (as the case may be) during the Income Year,
  - plus (c) Any amounts payable by the issuer or receivable by the holder (as the case may be) during the Income Year,
  - less (d) The Present Value (if any) of the financial arrangement as at the balance date on which the immediately preceding Income Year ended.

## 7 Example

### 1. Example A

- a) This is the same example as in Determination G3: Yield to Maturity Method (except for the dates). It is used to demonstrate that the methods give similar results.

On 12 March 1991 a holder acquires for \$1,012,500 the right to receive the following income:

	\$
15 May 1991	70,000
15 November 1991	70,000
15 May 1992	70,000
15 November 1992	1,070,000
	\$1,280,000

The holder balances on 31 March. All amounts are in New Zealand currency.

It will be found that the Annual Yield To Maturity Rate (R) is 16.2308% per annum; see Example A in Determination G10B: Present Value Calculation Methods.

- b) The present values at the end of each Income Year, calculated according to Determination G10B: Present Value Calculation Methods, are set out in the following schedule—

<u>Year ending 31 March</u>	1991	1992
Next period — End date	15/5/87	15/5/88
Present Value (A) (see note)	\$971,315	\$989,683
Payment at period end by issuer (B)	\$70,000	\$70,000
by holder (C)	—	—
Days from 31/3 to 15/5	45	45
$N = 365/45$	8.11111	8.11111
$F = R/(100 \times N)$	0.02001	0.02001
$R = 16.2308$		
Present Value = $(A + B - C)/(1 + F)$	\$1,020,887	\$1,038,895

NOTE: See example A in Determination G10B: Present Value Calculation Methods for these present values

- c) The following schedule may then be constructed, showing the income in respect of each Income Year—

<u>Income Year</u> ending 31 <u>March</u>	<u>Present Value</u> <u>at Year End</u> (a) or (d) \$	<u>Payment by</u> <u>Holder</u> (b) \$	<u>Payments</u> <u>by Issuer</u> \$	<u>Income</u> <u>Earned</u> <u>by</u> <u>Holder</u>	
1991	1,020,887	1,012,500	—	8,387	(i)
1992	1,038,895	—	140,000	158,008	(ii)
1993	—	—	1,140,000	101,105	(iii)
				<u>\$267,500</u>	

NOTE:

(i)  $\$1,020,887 - \$1,012,500 = \$8,387$

(ii)  $\$1,038,895 - \$1,020,887 + \$140,000 = \$158,008$

(iii) Calculated using the formula for the base price adjustment in Section 64F(2) of the Act:

$$a - (b + c)$$

Where

a =  $\$70,000 + \$70,000 + \$70,000 + \$1,070,000 = \$1,280,000$ , the sum of all amounts payable to the holder, and

b =  $\$1,012,500$ , the acquisition price, and

c =  $\$8,387 + \$158,008 = \$166,395$ , the amount of income derived to date by the holder.

Note that this is confirmed by extending the same calculation procedure used for 1991 and 1992, into 1993 as follows:—

a = 0, the Present Value at the end of the 1993 Income Year.

b = 0

c =  $\$1,140,000$ , the payments by the issuer in the year.

d =  $\$1,038,895$ , the Present Value at the previous balance date.

Hence

$$a - b + c - d = \$101,105.$$

## 2. Example B

- a) This example is also similar to that in Determination G3: Yield to Maturity Method (except for the dates).

On 12 March 1991 a holder acquires for \$1,012,500 the right to receive the following income:—

	\$
15 May 1991	70,000
15 November 1991	70,000
15 May 1992	70,000
15 November 1992	<u>1,070,000</u>
<b>Total</b>	<u><b>\$1,280,000</b></u>

The holder balances on 31 March. All amounts are in New Zealand currency.

This income would be typical of a New Zealand Government Stock with a 14% coupon maturing 15 November 1992.

Under Method B of calculating the Present Value of a financial arrangement, it is calculated that the Annual Yield To Maturity Rate is 16.265%. This is the interest rate at which the Present Value of payments due after 12 March 1991 is equal to \$1,012,500. See the foot note to this Example B for details of calculation using the HP-12C calculator.

- b) The present values at the end of each Income Year are calculated using Method B of Determination G10B: Present Value Calculation Methods. The method is the same as that adopted by the International Association of Bond Dealers and used in the HP-12C and similar calculators.

The calculation of present values in Example B may be made using the BOND PRICE function on the HP12C (or equivalent) calculator. The following steps reproduce the "Present Value at year end" for the Income Year ending 31 March 1991:

Specified rate	16.265	(g) (D.MY)	
Coupon % pa	14	(i)	
Value date	31.031987	(PMT)	
Maturity date	15.111988	(ENTER)	
Add accrued interest		(f) (PRICE)	96.824919
		(+)	102.084588

which is the per \$100 nominal price corresponding to \$1,020,846.



c) The following schedule may then be constructed:-

Income Year Ending 31 March	Present Value at Year End	Payment by Holder	Payments Issuer	Income Earned by Holder
	\$	\$	\$	\$
1991	1,020,846	1,012,500	—	8,346(i)
1992	1,039,241	—	140,000	158,395(ii)
1993	—	—	1,140,000	100,759(iii)
		Total		<u>\$267,500</u>

**Note:**

(i)  $\$1,020,846 - \$1,012,500 = \$8,346$

(ii)  $\$1,039,241 - \$1,020,846 + \$140,000 = \$158,395$

(iii) Calculated using the formula for the base price adjustment in Section 64F(2) of the Act.

$$a - (b + c)$$

Where:

a =  $\$70,000 + \$70,000 + \$70,000 + \$1,070,000 = \$1,280,000$ , the sum of all amounts payable to the holder,

b =  $\$1,012,500$ , the acquisition price, and

c =  $\$8,346 + \$158,395 = \$166,741$ , the amount of income derived to date by the holder.

Note that this is confirmed by extending the same calculation procedure used for 1991 and 1992, into 1993 as follows:—

a = 0, is the Present Value at the end of the 1993 Income Year.

b = 0

c =  $\$1,140,000$ , are the payments by the issuer in the year.

d =  $\$1,039,241$ , is the Present Value at the previous balance date.

Hence

$$a - b + c - d = \$100,759.$$

## Footnote:

The calculations may be made using the BOND PRICE function on the HP-12C (or equivalent) calculator.

## Calculating the Specified Discount Rate, R

The HP-12C assumes that the purchase price excludes accrued interest, whereas the actual purchase price of \$1,012,500 includes accrued interest from 15 November 1990 to 12 March 1991. This accrued interest is calculated as follows, per \$100 nominal:

Set up		(g) (D.MY)
Any YTM	0	(i)
Coupon Percent Pa.	14	(PMT)
Purchase date	12.031987	(STO)1
		(ENTER)
Maturity date	15.111988	(STO)2
		(f)(PRICE)
		(X > Y)4.524862

This accrued interest is then subtracted from the purchase price per \$100 nominal, of \$101.25, to give the ex-accrued interest purchase price.

Purchase price	101.25	(X > Y)
		(-)96.725138

The Specified Discount Rate (R) can then be calculated using the BOND YTM function;

Ex-accrued interest price	(PV)
Purchase date	(RCL)1
Maturity date	(RCL)2
	(f) (YTM) 16.265%

### 3. Example C

- a) This example illustrates the application of this determination to a straightforward perpetual security. It is identical to Example C in Determination G10B: Present Value Calculation Methods.

The perpetual has a face value of 100, a yearly coupon of 10% paid half yearly, and was issued at 78.00 on 1 August 1991. The issuer is a New Zealand resident taxpayer with a 30 June balance date.

First we have to calculate the Annual Yield to Maturity Rate. Although outside the scope of this determination this can be done as follows:

Since all payments are the same, Method A formula (i) of Determination G10B: Present Value Calculation Methods applies. The present value is 78.00 at the date of issue which is equal to:

$$\frac{E}{F}$$

where, E is the half yearly coupon, 5%. Therefore,

$$78.00 = \frac{5.00}{F}$$

whence

$$\begin{aligned} F &= \frac{5.00}{78.00} \\ &= 0.06410 \end{aligned}$$

and so the Specified Discount Rate in Determination G10B: Present Value Calculation Methods is

$$\begin{aligned} R &= 100 \times N \times F \\ &= 100 \times 2 \times 0.06410 \\ &= 12.82\% \end{aligned}$$

and this is the Annual Yield to Maturity Rate.

- b) Since the balance date is not a date on which an amount is payable, formula (ii) of Method A must be used to calculate the present value at the balance date, using the present value as at the immediately succeeding payment date and the payment then due.
- c) There are 32 days from 30 June to 1 August so that in all years—

$$N = \frac{365}{32}$$

$$\begin{aligned} F &= R/(100 \times N) \\ &= 12.82/(100 \times 365/32) \\ &= 0.01124 \\ 1 + F &= 1.01124 \end{aligned}$$

From Example C of Determination G10B: Present Value Calculation Methods, the present values at each balance date are calculated as follows:

Balance date	Next Period End		Present value	Payments by Issuer	Present Value at balance date (1)
	Date	A			
30/6/91	1/8/01	78.00	5.0	82.08	
30/6/92	1/8/92	78.00	5.0	82.08	
30/6/93	1/8/93	78.00	5.0	82.08	
.....	.....	.....	.....	.....	

Note: C = 0 in all cases (1) calculated as (A + B - C)/(1 + F)

1) Calculated as (A + B - C)/(1 + F)

d) The following schedule may then be constructed, showing the expenditure incurred by the issuer in respect of each income year:

Income year ending 30 June	Present value year end	Payments in year by—		Expenditure incurred by issuer
		Holder	Issuer	
	(a) or (d)	(b)	(c)	
1991	82.08	78.00	—	4.08 (i)
1992	82.08	—	10.00	10.00 (ii)
1993	82.08	—	10.00	10.00 (ii)
1994	82.00	—	10.00	10.00 (ii)
.....	.....	.....	.....	.....

Notes:

(i) 82.08 - 78.00 = 4.08

(ii) 82.08 + 10.00 - 82.08 = 10.00

The constant expenditure from 30 June 1992 income year onwards is to be expected, and would only change if the issuer's balance date changed, or there was a change in the terms of the security.

Unless the perpetual security is repaid under the terms of its issue (such as default), there will never be a base price adjustment.

#### 4. Example D

- a) This example illustrates the application of Method A to a more complicated perpetual note than in Example C. It is identical to Example D in Determination G10B: Present Value Calculation Methods.

The note was issued at 90.00 on 1 February 1991. It has a nominal capital of 100 and has coupon interest at 14% p.a. commencing on 1 August 1993 and payable halfyearly thereafter on 1 February and 1 August each year in perpetuity. The nominal capital is repayable only if the issuer defaults on a coupon payment. The issuer is a New Zealand resident taxpayer with a 30 June balance date.

The first step is to determine the Annual Yield To Maturity Rate. This must be done by trial and error.

The process is as follows:

- (i) Estimate F, the interest rate per period in decimal form. (For example, an Annual Yield To Maturity Rate of 14% payable quarterly gives  $F = 0.035$ ).
- (ii) Using Method A formula (i) of Determination G10B: Present Value Calculation Methods, calculate the present value as at the first payment date after which all amounts payable are the same, in this case as at 1/2/93.
- (iii) Then calculate the present value at the issue date, using Method A formula (ii) of Determination G10B. Often this can be accomplished quickly on a financial calculator.
- (iv) Compare this present value to the issue price and make a better estimate of F.
- (v) If F is not sufficiently accurate (generally ascertained by comparing it with the previous value, or comparing the present value to the issue price) go back to step (i).

In the present case, the following HP-12C program will enable successive estimates of F to be tested:

<u>Setup</u>		
4	(n)	Number of halfyear periods from issue to first coupon payment date
0	(PMT)	No coupon for 4 periods
90	(STO) 1	Save issue price
7	(STO) 2	Save coupon per period
<u>Program</u>		
(f)	(R/S)	to start
	(RCL)	2
	(RCL)	(i)
.01	(×)	
	(÷)	

This gives the present value E/F as at 1/2/93 of all payments after that date.

(FV)

(PV)

(RCL) 1

(+)

This gives the difference between the issue price and the present value.

(g) (GTO) 00

(f) (R/S) to end.

## Calculating

Estimate half yearly interest rate, press (i), then press (R/S).

<u>Estimate</u>	<u>Difference</u>
6.000	-2.411
6.250	2.118
6.150	0.352
6.140	0.172
6.130	-0.009
6.131	0.009
6.1305	0.000

Therefore the Annual Yield to Maturity Rate is  $2 \times 6.1305\% = 12.261\%$  p.a. Other programs might be needed in other examples.

- b) Since the balance date is not a date on which an amount is payable, formula (ii) of Method A must be used to calculate the present value as the balance date, using the present value as at the immediately succeeding payment date and the payment then due.
- c) There are 32 days from 30 June to 1 August so that in all years—

$$N = 365/32$$

$$F = R/(100 \times N) = 12.261/(100 \times 365/32) = 0.01075$$

$$1 + F = 1.01075$$

From Example D of Determination G10B: Present Value Calculation Methods, the present values at each balance date are calculated as follows:

<u>Balance date</u>	<u>Date</u>	<u>Next Period End</u>		<u>Present Value at balance date</u>
		<u>Present value</u>	<u>Payments by Issuer</u>	
		A	B	
30/6/91	1/8/91	95.52	—	94.50(i)
30/6/92	1/8/92	107.58	—	106.44(ii)
30/6/93	1/8/93	114.18	7.0	119.89(iii)
30/6/94	1/8/94	114.18	7.0	119.89
.....	.....	.....	.....	.....

(i) Calculated as  $(A + B - C)/(1 + F)$  where—

A = Present Value at the end of the period immediately following the given date

$$= 95.52$$

B = amounts receivable by the holder/payable by issuer at end of the period following the given date

$$= \text{nil}$$

C = amounts payable by holder/receivable by issuer at the end of the period immediately following the given date

$$= \text{nil (in all cases in this example)}$$

$$1 + F = 1.01075 \text{ (as above)}$$

$$\text{therefore } (A + B - C)/(1 + F) = (95.52 + 0 - 0)/1.01075 = 94.50$$

$$\text{(ii) } (A + B - C)/(1 + F) = 107.58 + 0 - 0/1.01075 = 106.44$$

$$(iii) (A + B - C)/(1 + F) = (114.18 + 7 - 0)/1.01075 = 119.89$$

- d) The following schedule may then be constructed, showing the expenditure incurred by the issuer in respect of each Income Year:

Income year ending 30 June	Present value at year end	Payments in year by—		Expenditure incurred by issuer
		Holder	Issuer	
	(a) or (d)	(b)	(c)	
1991	94.50	90.00	—	4.08 (i)
1992	106.44	—	—	11.94 (ii)
1993	119.89	—	—	13.45 (iii)
1994	119.89	—	14.00	14.00 (iv)
1995	119.89	—	14.00	14.00 (iv)
.....	.....	....	.....	.....

Notes:

- (i)  $94.50 - 90.00 = 4.50$   
(ii)  $106.44 - 94.50 = 11.94$   
(iii)  $119.89 - 106.44 = 13.45$   
(iv)  $119.89 + 14.00 - 119.89 = 14.00$

The constant expenditure from the 30 June 1994 income year onwards is to be expected, and would only change if the issuer's balance date changed, or there was a change in the terms of the security.

Unless the note is repaid under the terms of its issue (such as default) or sold, then there will never be a base price adjustment.

## About this document

General determinations set out the Commissioner's view on how the financial arrangements rules apply to a specific type of financial arrangement. All general determinations are binding on the Commissioner and some are also binding on taxpayers.